PARAMETRIC INSTABILITY IN A WEAKLY IONIZED MAGNETOACTIVE PLASMA

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The stability of a weakly ionized homogeneous plasma, situated in a weak superhigh frequency (shf) electric field and a constant magnetic field, is studied. Expressions are obtained for longitudinal wave deviation increments in the plasma, and for the threshold values of the external shf field, at which the system begins to develop instability. It is shown that the presence of the external magnetic field produces either a stabilizing or a destabilizing effect on the system, dependent on the orientation of the shf field. In particular, when the direction of the magnetic field **B** is perpendicular to the electric field **E** and the Langmuir electron frequency $\omega_{Le} = (4\pi n_e e_e^2/m_e)^{1/2}$ is less than the cyclotron electron frequency $\Omega_e = e_e B/m_e c$, the threshold value of shf electric field intensity in $(\omega_{Le}/\Omega_e)^3$ is lower than the corresponding value for an isotropic plasma.

In [1,2] it has been shown that even a weak shf field at frequencies $\omega_0 \sim \omega_{Le} = (4\pi n_e e_e^2/m_e)^{1/2}$ produces a parametric instability in a plasma with respect to deviation in potential disturbances. The presence of an external magnetic field effects a change in the basic state of the plasma situated in the shf field, as well as in the appearance of new spectra. Therefore it is natural to expect that the magnetic field will have a significant effect upon the threshold value of external shf field intensity, beginning with which excitation of the plasma occurs. In [3], in the study of parametric excitation of cyclotron waves in a highly ionized plasma, a conclusion was reached on the possibility of both a stabilizing and a destabilizing influence of the external magnetic field on the system, dependent on its orientation and magnitude. In the first part of this study, we will obtain the frequencies and increments of oscillation excitation by an external shf field for a weakly ionized homogeneous plasma. In the second part, the influence of an external magnetic field will be studied for the particular case of a shock plasma. It will be shown that aside from the stabilizing effect of the magnetic field, in some instances a decrease in threshold electric field intensity for instability, as compared to the corresponding value for an isotropic plasma, is possible.

1. We will examine the stability with respect to potential disturbances of a homogeneous weakly ionized plasma, situated in a constant homogeneous magnetic field **B** and an shf electric field **E** (t). We will assume the electric field to be homogeneous over the space

$$\mathbf{E}(t) = \mathbf{E} \sin \omega_0 t$$

and the electron distribution function to be Maxwellian.

In the presence of an intense external electric field, it is by no means necessary that the electron distribution function be Maxwellian. However, if the shf electric field frequency ω_0 is greater than the electron collision frequency ν_{en}, the distribution may be regarded as Maxwellian [4].

The dispersion equation for potential oscillations of a homogeneous plasma in a weak shf field for frequencies $\omega_0 \gg \omega$ has the form [5]:

$$\frac{1}{1+\delta\varepsilon_e^{(0)}} + \frac{1}{\delta\varepsilon_i^{(0)}} + \frac{1}{4}(kr_E)^2 f_r \left\{\frac{1}{\varepsilon^{(1)}} + \frac{1}{\varepsilon^{(-1)}}\right\} = 0 \tag{1.1}$$

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The following notation is used:

$$f_r = \left\{\frac{\omega_0^2\Omega_e^2}{\omega_0^2-\Omega_e^2}\sin^2\theta\sin^2\varphi + \left[\cos\theta\cos\chi_0 + \frac{\omega_0^2}{\omega_0^2-\Omega_e^2}\sin\theta\sin\chi_0\cos\varphi\right]^2\right\} \tag{1.2}$$

$$r_E = u_E/\omega_0, \quad u_E = e_eE/m_e\omega_0, \quad \varepsilon^{(n)} = 1 + \delta\varepsilon_e^{(n)} + \delta\varepsilon_i^{(n)}$$

where χ_0 is the angle between the vectors **B** and **E**; θ is the angle between **k** and **B**; φ is the angle between the planes **k** and **B**, and **E** and **B**; $\delta\varepsilon_a^{(n)} = \delta\varepsilon_a(n\omega_0 + \omega, \mathrm{k})$ is the partial longitudinal dielectric permeability, the derivation and expression for which may be found in [6].

Dispersion equation (1.1) at external field frequencies close to one of the electron hybrid frequencies ω_* has two solutions. The first corresponds to oscillations of the first harmonic of the nonequilibrium potential with hybrid frequencies

$$\begin{aligned}\omega^2 &= (\Delta\omega_0)^2 - \left(\frac{r_E}{r_{De}}\right)^2 f\frac{(\Delta\omega_0)\,\omega\omega_0\delta\omega_\pm}{(\delta\omega_\pm)^2+(\gamma_\pm)^2}\\ \gamma &= \gamma_0 - \frac{1}{2}\left(\frac{r_E}{r_{De}}\right) f\frac{\gamma_\pm\omega_0\Delta\omega_0}{(\delta\omega_\pm)^2+(\gamma_\pm)^2}\end{aligned} \tag{1.3}$$

and the second to oscillations of the zeroth harmonic

$$\begin{aligned}\omega^2 &= (\omega_\pm)^2 + 2\left(\frac{r_E}{r_{De}}\right)^2 f\frac{\Delta\omega_0\left[(\Delta\omega_0)^2+\gamma_0^2-\omega_\pm^2\right]\omega_0\omega_\pm^2}{\left[(\Delta\omega_0)^2+\gamma_0^2-\omega_\pm^2\right]^2-4(\omega_\pm\gamma_0)^2}\\ \gamma &= \gamma_\pm - 2\left(\frac{r_E}{r_{De}}\right)^2 f\frac{\Delta\omega_0\gamma_0\omega_0\omega_\pm}{\left[(\Delta\omega_0)^2+\gamma_0^2-(\omega_\pm)^2\right]^2+4(\omega_\pm\gamma_0)^2}\end{aligned} \tag{1.4}$$

wherein the following notation is used:

$$\begin{aligned}\Delta\omega_0 &= \omega_0 - \omega_*\left[1+\delta\varepsilon_{eT}'(\omega_*)\left(\omega_*\frac{\partial\delta\varepsilon_e'(\omega_*)}{\partial\omega_*}\right)^{-1}\right]\\ \delta\omega_\pm &= \omega-\omega_\pm\\ \gamma_0 &= \frac{\nu_{en}}{2}\frac{3\omega_0^2-2\omega_{Le}^2-\Omega_e^2}{2\omega_0^2-\omega_{Le}^2-\Omega_e^2} + \frac{\omega_0^2\omega_{Le}^2(\omega_0^2-\Omega_e^2)}{2\omega_0^2-\omega_{Le}^2-\Omega_e^2}\\ &\times\frac{\sqrt{\pi}}{(kv_{Te})^3}\left[\frac{\omega_0^2(\omega_{Le}^2-\Omega_e^2-\omega_0^2)}{\omega_{Le}^2\Omega_e^2}\right]^{-1/2}\exp\left\{-\left(\frac{\omega_0}{kv_{Te}}\right)^2\frac{\omega_{Le}^2\Omega_e^2}{\omega_0^2(\omega_{Le}^2+\Omega_e^2-\omega_0^2)}\right\}\end{aligned} \tag{1.5}$$

$$\begin{aligned}\gamma_\pm &= \frac{\nu_{in}}{2}\frac{3\omega^2-2\omega_s^2-\Omega_i^2}{2\omega^2-\omega_s^2-\Omega_i^2} + \frac{\sqrt{\pi}}{(kv_{Ti})^3}\left[\frac{\omega_s^2\Omega_i^2}{\omega^2(\omega_s^2+\Omega_i^2-\omega^2)}\right]^{1/2}\frac{\omega^2\omega_s^2(\omega^2-\Omega_i^2)}{2\omega^2-\omega_s^2-\Omega_i^2}\\ &\times\left[\frac{\omega_{Le}^2}{\omega_{Li}^2}\frac{v_{Ti}^3}{v_{Te}^3}+\exp\left\{-\left(\frac{\omega_0}{kv_{Ti}}\right)^2\frac{\omega_s^2\Omega_i^2}{\omega^2(\omega_s^2+\Omega_i^2-\omega^2)}\right\}\right]\end{aligned} \tag{1.6}$$

where $\delta\varepsilon_{aT}'$ is the thermal addition to the partial dielectric permeability tensor

$$\delta\varepsilon_a = \delta\varepsilon_a' + \delta\varepsilon_{aT}' + i\delta\varepsilon_a''$$

and finally

$$f = {}^1/_8 f_e f_i f_z \tag{1.7}$$

where

$$\begin{aligned}f_e &= \frac{\omega_0^2-\Omega_e^2}{2\omega_0^2-\omega_{Le}^2-\Omega_e^2}, \quad f_i = \frac{\omega^2-\Omega_i^2}{2\omega^2-\omega_s^2-\Omega_i^2}\\ \omega_\pm^2 &= \frac{1}{2}\left\{(\omega_s^2+\Omega_i^2)\pm\sqrt{(\omega_s^2+\Omega_i^2)^2-4\omega_s^2\Omega_i^2\frac{\omega_0^2(\omega_{Le}^2+\Omega_e^2-\omega_0^2)}{\omega_{Le}^2\Omega_e^2}}\right\}\end{aligned} \tag{1.8}$$

$$\omega_s = \frac{kv_s}{\sqrt{1+(kr_{De})^2}}, \quad v_s = \left(\frac{T_e}{m_i}\right)^{1/2}, \quad r_{De} = \frac{v_{Te}}{\omega_{Le}}, \quad v_{Te} = \left(\frac{T_e}{m_e}\right)^{1/2}, \quad \omega_{Le} = \left(\frac{4\pi n_e e_e^2}{m_e}\right)^{1/2}$$

with ν_{an} being the collision rate of charged particles with neutrals.

Equations (1.5) and (1.6) describe the excitation of hybrid electron oscillations and low frequency oscillations with cold magnetized $[(k_\perp v_{Ti})^2/2\Omega_i^2 < 1]$ ions, and of hot plasma electrons with the external shf field. If we set $f = 1/8$ in Eqs. (1.5) and (1.6), we obtain the formula which describes parametric excitation of a periodic instability in an isotropic plasma [1,2]. The expressions for excitation of a weakly ionized plasma with nonmagnetized ions $[(k_\perp v_{Ti})^2/2\Omega_i^2 > 1]$ by an shf field are obtained from Eqs. (1.5) and (1.6) if $f_i = 1$.

2. We will examine the threshold values of external shf field intensity, above which the system becomes unstable. At first, we will consider the case of small high frequency increments $(\gamma_0 < \omega)$. It is easy to see that the lowest shf field values are attained in the decay cases $\Delta\omega_0 = \omega_\pm$. Assuming the increment in Eqs. (1.5) and (1.6) to be equal to zero, we obtain the threshold shf field intensity

$$\lambda^\pm = \left(\frac{r_E}{r_{De}}\right)^2 = \frac{2}{f}\frac{\gamma_\pm\gamma_0}{\omega\omega_0} \tag{2.1}$$

We will limit our analysis to the collision case, when $\gamma_\pm$ and γ_0 are determined by the collision of charged particles with neutrals. We will rewrite the threshold intensity for the collision case in the more convenient form

$$\lambda^\pm = 4K/\nu_{en}\nu_{in}/\omega_s\omega_{Le} \tag{2.2}$$

where

$$K = K_e K_i f_r^{-1} \tag{2.3}$$

$$K_i = \frac{\omega_s}{\omega}\frac{3\omega^2 - 2\omega_{Li}^2 - \Omega_i^2}{\omega^2 - \Omega_i^2} \tag{2.4}$$

$$K_e = \frac{\omega_{Le}}{\omega_0}\frac{3\omega_0^2 - 2\omega_{Le}^2 - \Omega_e^2}{\omega_0^2 - \Omega_e^2} \tag{2.5}$$

For the case of a sufficiently great high-frequency increment $(\gamma_0 > \omega)$, for the collision limit the threshold value at optimum frequency deviation $\Delta\omega_0 = \gamma_0/\sqrt{3}$ has the form

$$\lambda^\pm = \frac{8}{3\sqrt{3}} K \frac{\nu_{en}^2\nu_{in}}{\omega_{Le}^2\omega_s} \tag{2.6}$$

The quantity K herein is determined by Eq. (2.3), in which K_i and f_r, as before, are expressed by means of Eqs. (2.4) and (1.2), respectively. The electronic portion of the coefficient K, in this case, has the form

$$K_e = \frac{\omega_{Le}}{\omega_0}\frac{(3\omega_0^2 - 2\omega_{Le}^2 - \Omega_e^2)^2}{(\omega_0^2 - \Omega_e^2)(2\omega_0^2 - \omega_{Le}^2 - \Omega_e^2)} \tag{2.7}$$

Equations (2.2)-(2.7) determine the threshold for a plasma with magnetized ions. In the case of unmagnetized ions one must assume $K_i = 1$ in Eqs. (2.2)-(2.7). For a plasma without a magnetic field the coefficient K = 1. Thus, the effect of a magnetic field on parametric instability is expressed through the coefficient K, since by its dependence on the magnetic field intensity one may evaluate the change in threshold value as compared to the corresponding value for an isotropic plasma.

It is known that in a plasma with magnetized cold ions and hot electrons there exist two branches of weakly damped oscillations [see Eq. (1.8)]. In the case of a strongly nonisotropic plasma $(\omega_s > \Omega_i)$ for the spectrum branch of frequency $\omega_+ = \omega_s$ the threshold value of the shf field is determined by Eqs. (2.2) and (2.6), with $K_i = 1$. As can be shown, the threshold value for this branch is $(\Omega_i\omega_s^{-1})^4$ times smaller than the corresponding value for the other branch of this spectrum

$$\omega_-^2 = \Omega_i^2\frac{\omega_0^2(\omega_{Le}^2 + \Omega_e^2 - \omega_0^2)}{\omega_{Le}^2\Omega_e^2}$$

that is, for a drop in frequency $(\omega_+ > \omega_-)$ the threshold increases.

We will examine the converse case $(\omega_s < \Omega_i)$ of longwave oscillations. The coefficients K_i of the respective oscillation branches take on the form

$$K_i(\omega_+) = \frac{\Omega_i}{\omega_s}\left[\frac{\omega_{Le}^2\Omega_e^2}{(\omega_0^2-\omega_{Le}^2)(\omega_0^2-\Omega_e^2)}\right] \quad (2.8)$$

$$K_i(\omega_-) = \left[\frac{\omega_{Le}^2\Omega_e^2}{\omega_0^2(\omega_{Le}^2+\Omega_e^2-\omega_0^2)}\right]^{1/2} \quad (2.9)$$

Thus, as follows from Eqs. (2.2), and (2.3), the threshold intensities of the respective spectra are distinguished by the values of the coefficients in Eqs. (2.8) and (2.9). In particular, when the external field frequency approaches either the maximum ($\omega_0^2 \approx \omega_{Le}^2 + \Omega_e^2$) or minimum ($\omega_0^2 \ll \omega_{Le}^2, \Omega_e^2$) hybrid frequency, the threshold for the upper spectrum branch approaches a minimum [Eq. (1.8)]. Analogously, for external field frequencies in the region of either $\omega_0^2 \approx \omega_{Le}^2$ or $\omega_0^2 \approx \Omega_e^2$, the threshold for the upper spectrum branch [Eq. (1.8)] rises sharply in comparison to the lower.

We will examine two limiting cases, wherein either the first term in square parentheses in Eq. (1.2) can be neglected (transverse orientation), or the first term is the largest (longitudinal orientation). The electronic portion of the coefficient K for the cases indicated, for example in decay instability, can be written as

$$K_e f_r = \frac{\omega_{Le}^3}{\omega_0^3}\,\frac{3\omega_0^2-2\omega_{Le}^2-\Omega_e^2}{\omega_0^2-\omega_{Le}^2}\,\{[\sin^2\varphi+\omega_0^2\Omega_e^{-2}\cos^2\varphi]\sin^2\chi_0\}^{-1} \quad (2.10)$$

$$K_e f_r = \frac{\omega_{Le}}{\omega_0}\,\frac{3\omega_0^2-2\omega_{Le}^2-\Omega_e^2}{\omega_0^2-\Omega_e^2}\left[\frac{\omega_{Le}^2\Omega_e^2}{\omega_0^2(\omega_{Le}^2+\Omega_e^2-\omega_0^2)}\right]\cos^{-2}\chi_0 \quad (2.11)$$

From the equation for the case of transverse orientation, Eq. (2.10), it follows that at external field frequencies close to either the maximum ($\omega_0^2 \approx \omega_{Le}^2 + \Omega_e^2$) or minimum ($\omega_0^2 \ll \omega_{Le}^2, \Omega_e^2$) electron hybrid frequencies, for a system with $\omega_{Le} < \Omega_e$, the threshold field is reduced by a factor of $(\omega_{Le}, \Omega_e^{-1})^3$ in comparison with the corresponding value for an isotropic plasma. We note also, that for the frequency ranges $\omega_0^2 \approx \omega_{Le}^2 + \Omega_e^2$ and $\omega_0^2 \ll \omega_{Le}^2, \Omega_e^2$, in the case of longitudinal orientation [Eq. (2.11)], the coefficient $K \gg 1$, i.e., the magnetic field produces a strong stabilizing effect on the system. The stabilizing effect of the magnetic field can also be obtained because of the corresponding orientation of the external shf field. For example, in the case of longitudinal orientation for $\omega_0 \approx \Omega_e$, as follows from Eq. (2.11), the effect is produced due to increase in $\cos^{-2}\chi_0$. The threshold for nondecay instability possesses similar properties.

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